the revolution. In steady conditions, ΔT is 13 K at the end of irradiation while at the end of the revolution $\Delta T = -12$ K, i.e., changes sign.

Characteristic curves of the variation in temperature difference over the thickness of the composite shell in steady temperature conditions ($T_1 > T_{av}$) after one revolution are shown in Fig. 3. When $\delta = 20$ mm, the temperature at the end of the revolution increases over a depth approximately equal to 0.25 of the thickness away from the external shell surface, and then decreases to the shell–mandrel contact boundary, i.e., the temperature profile over the composite thickness takes the form of a curve with a maximum close to the outer irradiated surface.

As is evident from Fig. 3 (curves 5 and 6), in steady conditions, in a shell of thickness $\delta = 2$ mm, the temperature variation occurs in the course of a revolution over the whole thickness. With increase in δ and ω, the limit of the temperature variations is shifted to the outer surface.

Analysis of the numerical experiments shows that the model proposed permits the choice of optimal technological and constructional parameters ensuring the heat-treatment conditions of composite materials with permissible temperature differences, especially if it is difficult to perform experimental temperature measurements, for example, as in the case of mandrel revolution here considered. Thus, using well-founded methods of energy supply and heat-treatment conditions of the wound components, directional influence may be exerted on the qualitative characteristics of composite materials, within known limits.

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POSITIVE COLUMN OF GLOW DISCHARGE IN LONGITUDINAL GAS FLOW

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A system of nonlinear equations describing a positive discharge column in a cylindrical channel with a gas flow is solved.

It is known that a gas flow has a positive influence on the characteristics of a glow discharge (GD): It allows the energy contribution to the discharge to be increased and ensures high stability [1-3]. In connection with this, the development of methods of calculating GD characteristics in a gas flow is an urgent problem. In the present work, an analytical solution of the problem of the positive column (PC) of a discharge is obtained, taking into account that the processes are nonsteady.

Suppose, as in the theory of a discharge with no flow in diffuse conditions [4], that the electron temperature is constant over the channel cross section, ionization occurs from the ground state of the atom by single-electron impact, and volume recombination, diffusion along PC, and convective transfer in the radial direction may be neglected in comparison with ambipolar diffusion, volume ionization by electron impact, and convective transfer along the channel axis. The degree of ionization in the PC region is sufficiently small ($n_e/N \leq 10^{-5}$); therefore, the frequency of collisions between charged particles is considerably less than the collision frequency of charged particles and neutral components of the gas. In connection with this, the thermal conductivity, specific heat, and gas density are determined basically by the properties of neutral particles [5]. These simplifying assumptions for the case of weakly ionized GD plasma are physically justified, have been discussed

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repeatedly, and were used earlier in [3-7]. Then the GD positive column in a longitudinal laminar gas flow may be described by the equations

$$\omega\frac{\partial n_e}{\partial t}+\frac{V}{l}\frac{\partial n_e}{\partial z}=\frac{D_a}{R^2 r}\frac{\partial}{\partial r}\left(r\frac{\partial n_e}{\partial r}\right)+\nu_e n_e, \tag{1}$$

$$I(t)=2\pi R^2 e b_e E(z,t)\int_0^{\xi} n_e(r,z,t)\,r\,dr, \tag{2}$$

$$\frac{\partial S}{\partial t}+a\frac{\partial S}{\partial z}=\frac{a\gamma}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}S\right)+\alpha n_e E^2 \tag{3}$$

under the conditions

$$\begin{gathered}n_e|_{r=\xi}=0,\ \ n_e|_{z=0}=\varphi_1(r,t),\ \ n_e|_{t=0}=\varphi_{11}(r,z),\\ S|_{r=\xi}=0,\ \ S|_{z=0}=\varphi_2(r,t),\ \ S|_{t=0}=\varphi_{22}(r,z),\\ \left.\frac{\partial n_e}{\partial r}\right|_{r=0}=0,\ \ \left.\frac{\partial S}{\partial r}\right|_{r=0}=0,\ S=T-T_R,\ \xi=\xi_0\sqrt{1+kz}.\end{gathered} \tag{4}$$

Here $a = V/\ell\omega$, $\delta = D_a\ell/VR^2$, $\alpha = k\xi_0/\delta$, $\gamma = \varkappa\ell/\rho c_p VR^2$.

The ionization frequency ν_e is a function of E/p [3]. Taking the small variation in pressure p along the PC in GD conditions into account [4], ν_e is assumed to be a function of E, and it is supposed that

$$\nu_e=\beta E^m. \tag{5}$$

The parameters β and m are determined by the pressure and nature of the gas. The correctness of this approximation was proven in [8] and, when m = 2, used in [7].

The solution of Eq. (1) in the new coordinate system $r_1 = r/\xi$, $x = z$, $\tau = t - z/a$ is sought in the form

$$n_e(r_1,x,\tau)=u(r_1,x,\tau)\exp\left(\frac{\beta}{a\omega}\int_0^x E^m dx\right). \tag{6}$$

Integration of Eq. (1) gives

$$\begin{gathered}n_e(r_1,x,\tau)=\sum_{n=1}^{\infty} f_n(\tau)\exp\left[\frac{\beta}{a\omega}\int_0^x\left(E^m-\frac{a\omega\mu_n^2\delta}{\beta\xi^2}\right)dx\right]\Phi_n(\mu_n r),\\ \Phi_n(\mu_n r)=\Phi_n\left(\frac{\mu_n^2\delta}{k\xi_0^2},\ 1,\ -\frac{k\xi_0^2}{4\delta}r_1^2\right),\end{gathered} \tag{7}$$

where μ_n are the roots of the equation $\Phi(\mu) = 0$. The function $f_n(\tau)$ is determined from the condition in Eq. (4)

$$\begin{gathered}f_n(t-z/a)=\begin{cases}A_n(t-z/a), & \tau>0,\\ B_n(z-at)\,K^{-1}(z-at), & \tau\leqslant 0;\end{cases}\\ K(z)=\exp\left(-\delta\int_0^z\frac{\mu_n^2}{\xi^2}dx\right)\exp\left[\frac{\beta}{a\omega}\left(\frac{I(0)}{2\pi R^2 e b_e}\right)^m\right]\int_0^z\left(\int_0^{\xi}\varphi_2(r,z)\,r\,dr\right)^{-m}dx.\end{gathered} \tag{8}$$

The general solution of Eq. (1) satisfying Eq. (4) takes the form

$$n_e(r,z,t)=\frac{I(t)}{2\pi R^2\xi^2 e b_e E}\left(\frac{\sum\limits_{n=1}^{\infty}A_n(t-z/a)\exp\left(-\delta\int_0^z\frac{\mu_n^2}{\xi^2}dx\right)\Phi_n(\mu_n r)}{\sum\limits_{n=1}^{\infty}A_n(t-z/a)\exp\left(-\delta\int_0^z\frac{\mu_n^2}{\xi^2}dx\right)\gamma_n}+\right. \tag{9}$$

$$+\frac{\sum_{n=1}^{\infty} B_n(z-at)\exp\left(\delta\int_z^{(z-at)}\frac{\mu_n^2}{\xi^2}dx\right)\Phi_n(\mu_n r)}{\sum_{n=1}^{\infty} B_n(z-at)\exp\left(\delta\int_z^{(z-at)}\frac{\mu_n^2}{\xi^2}dx\right)\gamma_n}\Bigg),\quad \gamma_n=\int_0^1\Phi_n(\mu_n r)\,r dr. \tag{9}$$

Here $A_n(t)$, $B_n(z)$ are the Fourier coefficients in the expansion of φ_1 and φ_2 with respect to the functions $\Phi_n(\mu_n r)$ in the interval $0 \le r \le \xi$, satisfying the conditions $A_n(t) = 0$ when $\tau \le 0$ and $B_n(z) = 0$ when $\tau > 0$.

The solution of nonlinear Eq. (2) for the electric field strength is as follows:

$$E(z,\ t)=\Psi(z,\ t)\left\{\frac{m\beta}{a\omega}\int_0^z\Psi^m(z,\ t)\,dz+1\right\}^{-\frac{1}{m}},$$

$$\Psi(z,\ t)=I(t)\left[2\pi R^2\xi^2 eb_e\sum_{n=1}^{\infty}f_n(t-z/a)\exp\left(-\delta\int_0^z\frac{\mu_n^2}{\xi^2}dx\right)\gamma_n\right]^{-1}. \tag{10}$$

Substitution of the formulas obtained for $E(z, t)$ and $n_e(r, z, t)$ into Eq. (3) allows the distribution of the power density of internal heat sources $q(r, z, t) = \chi\delta E^2$ to be determined and the temperature field of neutral particles in the channel to be calculated, taking account of variability of the PC radius. Integration of Eq. (3) for the case when $\tau > 0$ gives

$$S(r_1,\ z,\ t)=\sum_{i=1}^{\infty}(1+kz)^{-4\mu_i^2\gamma/k}\left\{X_i(t-z/a)+\frac{2\alpha}{H_i}\int_0^z\int_0^1 n_e E^2\times\right.$$

$$\left.\times\Phi_i(\mu_i r_1)(1+kz)^{4\mu_i^2\gamma/k}\exp\left(\frac{k\xi_0^2}{4\gamma}r_1^2\right)r_1dr_1dz\right\}\Phi_i(\mu_i r_1), \tag{11}$$

$$H_i=\int_0^1\Phi_i^2(\mu_i r)\exp\left(\frac{k\xi_0^2}{4\gamma}r_1^2\right)r_1dr_1,$$

where $X_i(t)$ are the Fourier coefficients in the expansion of φ_2 with respect to the degenerate hypergeometric functions $\Phi_i(\mu_i r_1)$ in the interval $0 \le r_1 \le 1$. Equations (9)-(11) form the accurate solution of the system in Eqs. (1)-(3), and describe the distribution of electron concentration, electric field strength, and gas temperature in the axisymmetric PC of a nonsteady GD, taking account of variability of the column radius. As is evident from the solution, the distribution of the discharge characteristics in the PC is of wave type. In the PC region up to the wavefront of the perturbation ($\tau \le 0$), the discharge characteristics are determined by their initial distriblution. In the region of developed instability ($\tau > 0$), the influence of the initial ($t = 0$) distribution of the PC characteristics vanishes and $n_e(r, z, t)$, $E(z, t)$, $T(r, z, t)$ are determined by the functions $\varphi_1(r, t)$ and $\varphi_2(r, t)$.

To simplify the analysis, particular cases are considered. Suppose that $\varphi_1(r, t) = A_1\Phi_1(\mu_1 r)$, $I = I_0(1 + i\cos t)$, $i = I_m/I_0 \ll 1$, $\varphi_2(r, t) = X_1\Phi_1(\mu_1 r)$ – the case when small perturbations are imposed on the dc GD.

In the limiting case as $\omega \to \infty$, it follows from Eqs. (9) and (10) when $\tau > 0$ for the cylindrical section of the column that

$$n_e(r,\ z,\ t)=A_1E_0\left(\frac{\beta R^2\xi_0^2}{\lambda_1 D_a}\right)\left\{1-\left(1-\frac{\lambda_1^2 D_a}{\xi_0^2R^2\beta E_0^2}\right)\exp\left(-\frac{2\lambda_1^2\delta}{\xi_0^2}z\right)+\right.$$

$$\left.+\frac{i^2}{2}\left[1-\exp\left(-\frac{2\lambda_1^2\delta}{\xi_0^2}z\right)\right]\right\}^{\frac{1}{2}}J_0(\lambda_1 r). \tag{12}$$

Here λ_1 is the first root of the equation $J_0(\lambda) = 0$, $E_0 = I_0/2\pi R^2 eb_e A_1\gamma_1$, $m = 2$.

As is evident from Eq. (12), the electron concentration cannot change markedly within the oscillation period in the case of a high-frequency perturbation field. In the case of a GD when small perturbations may be neglected, it follows from Eq. (12) that, as $\ell/V \to \infty$, the electron concentration tends to its limiting value

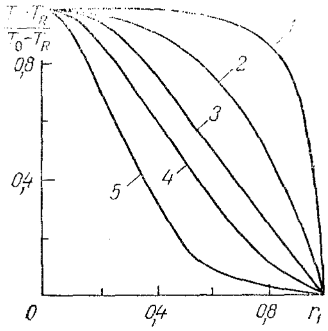

Fig. 1

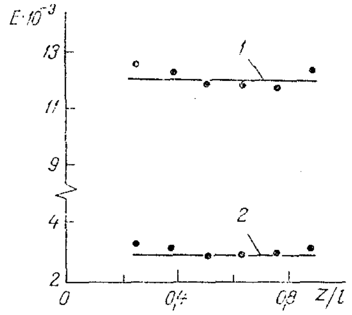

Fig. 2

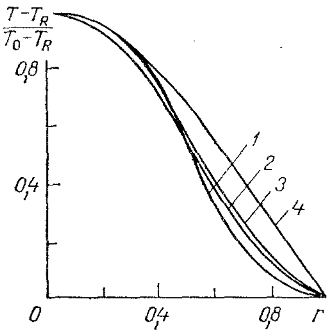

Fig. 3

Fig. 1. Distribution of relative temperature over the radius for various Ω: 1) Ω = −30; 2) −10; 3) 0; 4) 10; 5) 30.

Fig. 2. Distribution of electric field strength along the PC for gases: 1) nitrogen, p = 1.3 kPa, I = 0.015 A; 2) helium, 1.3 kPa, 0.025 A. The points correspond to experiment and the curves to calculation. E, $V \cdot M^{-1}$.

Fig. 3. Distribution of gas temperature over the radius for helium (1) and nitrogen (2) when p = 1.3 kPa, I = 0.025 A, V = 5 $m \cdot sec^{-1}$; 3) zero-order Bessel function; 4) calculation results.

$$\lim_{l/V\to\infty} n_e = n_{e\infty} = \frac{I_0 J_0(\lambda_1 r)}{2\pi R^2 \xi_0^2 e b_e \gamma_1 E_\infty}. \tag{13}$$

The limiting value of the electric field strength for arbitrary m takes the form

$$E_\infty = \left(\frac{\lambda_1^2 D_a}{\xi_0^2 R^2 \beta}\right)^{\frac{1}{m}}. \tag{14}$$

Thus, in the GD positive column, there is a section where the electron concentration and the electric field strength does not depend on the longitudinal coordinate, and is determined by Eqs. (13) and (14). In the more particular case when m = 2, $\xi_0 = 1$, and $i \simeq 0$, Eqs. (12)-(14) agree with the results of [4, 7].

The temperature field in the discharge channel is largely determined by the distribution of internal heat sources q(r, z, t). As follows from Eq. (11), the influence of internal heat sources on the temperature distribution decreases with increase in the rate of gas injection, and in the limiting case when V increases infinitely the temperature field is basically determined by the temperature distribution in the initial cross section

$$S \simeq X_1 \Phi_1 \left(\frac{\mu_1^2 \gamma}{\xi_0^2 k},\ 1,\ -\frac{k\xi_0^2}{4\gamma}\,\frac{r^2}{\xi^2}\right), \quad \Omega = \frac{\xi_0^2 k}{\gamma}. \tag{15}$$

Temperature profiles calculated with various Ω are shown in Fig. 1. As is evident, when $k < 0$ ($\Omega < 0$), increase in V leads to expansion of the temperature profile and conversely, in the case when $k > 0$ ($\Omega > 0$), to compression. Thus, in an expanding PC, increase in gas velocity leads to narrowing of the temperature and in a narrowing PC to expansion.

With the aim of comparing the theoretical results with experimental data, the distribution of electric field strength and the temperature in a chamber of cylindrical geometry are investigated. The experimental apparatus and measurement procedure were described in [9]. Analysis of the results of probe measurements shows that the electric field strength along the PC is practically constant. Thus, the most extended section of the PC is the limiting section.

Results of calculating the limiting value E_∞ according to Eq. (14) and experimental data for helium and nitrogen are shown in Fig. 2. The following approximations are used in calculating E_∞: for helium, $\nu_e = 0.37E^3$; for nitrogen, $\nu_e = 10^{-8}E^6$; ν_e, sec^{-1}. The approximations adopted for ν_e as a function of E in the given range of variation in the external parameters agrees with the results of numerical calculations and experiments [3, 4, 10]. As is evident from Fig. 2, the results of calculating E_∞ are in satisfactory agreement with experimental data.

Experiments show that the influence of the geometry of the initial PC section is significant. In a discharge chamber with rod electrodes along the channel axes, an expanding initial section ($k > 0$) is established and, with increase in gas velocity, narrowing of the temperature profile T(r) is observed. In the case when the hollow electrode positioned upstream ensures a constricting initial section ($k < 0$), growth in V leads to expansion of the temperature profile. Experimental data and the results of calculating T are shown in Fig. 3. As is evident, the radial distribution of gas temperature in the PC of a discharge with a longitudinal gas flow considerably deviates from Bessel form. Taking account of the geometry of the initial section (curve 4) leads to the best agreement between theory and experiment.

The results obtained may be useful in engineering calculations of gas-discharged flow chambers.

NOTATION

I, current; E, electric field strength; σ, electrical conductivity; n_e, electron concentration; N, concentration of neutral particles; b_e, electron mobility; e, electron charge; V, mean-mass gas velocity; D_a, ambipolar diffusion coefficient; ν_e, ionization frequency; T_R, wall temperature of channel; T, gas temperature; χ, coefficient characterizing the proportion of energy consumed in gas heating; ϰ, thermal conductivity; c_p, specific heat; p, ρ, gas pressure and density; R, tube radius; ℓ, column length; r, z, cylindrical coordinates referred to R and ℓ; ξ_0, dimensionless column radius; t_1, time; $t_1\omega = t$.

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